PEDAL CURVES AND SMARANDACHE CURVES OBTAINED FROM DARBOUX VECTOR BELONGING TO THE FRENET-LIKE FRAME OF A POLYNOMIAL SPACE CURVE

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Absract: In this study, firstly, the pedal curve was defined that is the geometric locus of the perpendicular projections onto the Darboux vector obtained from the Flc frame $\{T, D_2, D_1\}$. The curvature and torsion was calculated by expressing Frenet vectors T_1, N_1, B_1 of pedal curves in terms of Flc frame vectors T, D_2, D_1. Next, the Smarandache curves were defined by taking Frenet vectors of pedal curves as the position vectors. Finally, for each Smarandache curve, expressions of the polynomial curve based on Flc vectors and curvatures were obtained by calculating Frenet vectors, curvatures and torsions Thus, new pedal curves were added to the curves family by using new frame vectors.

Keywords: Darboux pedal curve, Flc-pedal curve, Smarandache curves

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1.Basic Concepts

In differential geometry, frames and the applications of frames to curves and surfaces have an important place. Frenet-like frame (or Flc frame) is a variation of Frenet frame and is a mathematical apparatus used especially, when working on polynomial type space curves. This frame, was developed to extend the limitations of Frenet frame and better describe the geometric properties of curves. By Flc frame, the number of singular points of a curve in Frenet frame can be reduced. Using this frame, it is effective in planning polynomial trajectories in Frenet coordinates and converting these trajectories into Cartesian coordinates. For deatiled information, see: [9 – 11]. When the Frenet vectors of a curve are taken as position vectors, the regular curves drawn by these vectors are called Smarandache curves [8]. Some properties of Smarandache curves were examined by defining Smarandache curves in Euclidean Space [1],[17]. The geometric locus of perpendicular projection points onto the tangent or normal

vector of a curve from a point not on the curve is defined as pedal (or contra pedal) curve. Some of the studies on such curves are given in the sources, $[3-6]$, $[12-16]$, $[18-24]$.

In previous conferences, the pedal curves and applications observed on osculator plane (e.g. [25]), normal plane (e.g. [26]), rectifying plane (e.g. [27]), osculator-like plane (e.g. [28]) and normal-like plane (e.g. [29]) were previously presented by Şenyurt and Ertem Kaya. This document differs from previous four conference full papers is the pedal curves and applications observed on rectifying-like plane.

In 3-dimensional Euclidean Space E^3, Frenet vectors, curvatures and Frenet formulae of a differentiable curve $\alpha(t)$ are given as following equalities, respectively.

$$T=\frac{\alpha'}{\|\alpha'\|}, N=B\wedge T=\frac{(\alpha'\wedge\alpha'')\wedge\alpha'}{\|\alpha'\wedge\alpha''\|.\|\alpha'\|}, B=\frac{\alpha'\wedge\alpha''}{\|\alpha'\wedge\alpha''\|}, \tag{1.1}$$

$$\kappa=\frac{\|\alpha'\wedge\alpha''\|}{\|\alpha'\|^3}, \quad \tau=\frac{\det(\alpha',\alpha'',\alpha''')}{\|\alpha'\wedge\alpha''\|^2}, \tag{1.2}$$

$$T'=v\kappa N, \ N'=v(-\kappa T+\tau B), \ B'=-v\tau N \tag{1.3}$$

where $v=\|\alpha'\|$, $[2],[7]$.

Let α be a nth degree regular polynomial curve in E^3. Frenet-like (Flc) vectors, Frenet-like curvatures, Frenet-like derivative formulas obtained from this vector are given following equalities, respectively $[9-11]$..

$$T=\frac{\alpha'}{\|\alpha'\|}, \ D_1=\frac{\alpha'\wedge\alpha^{(n)}}{\|\alpha'\wedge\alpha^{(n)}\|}, \ D_2=D_1\wedge T, \tag{1.4}$$

$$\begin{aligned} d_1&=\frac{\langle T',D_2\rangle}{v}=\frac{\langle\alpha'\wedge\alpha'',\alpha'\wedge\alpha^{(n)}\rangle}{\|\alpha'\|^3.\|\alpha'\wedge\alpha^{(n)}\|},\\ d_2&=\frac{\langle T',D_1\rangle}{v}=\frac{\langle\alpha''\wedge\alpha',\alpha^{(n)}\rangle}{\|\alpha'\|^2.\|\alpha'\wedge\alpha^{(n)}\|},\\ d_3&=\frac{\langle D_2',D_1\rangle}{v}=\frac{\langle\alpha'\wedge\alpha'',\alpha^{(n)}\rangle.\langle\alpha',\alpha^{(n)}\rangle}{\|\alpha'\|^2.\|\alpha'\wedge\alpha^{(n)}\|^2}, \end{aligned} \tag{1.5}$$

$$T'=v(d_1D_2+d_2D_1), \ D_2'=v(-d_1T+d_3D_1), \ D_1'=v(-d_2T-d_3D_2), \tag{1.6}$$

where if we take $\varphi = \measuredangle(ND_2)$, then we get following equalites.

$$\cos\varphi = \langle D_2, N\rangle = \langle D_1, B\rangle = \frac{\langle \alpha' \wedge \alpha'', \alpha' \wedge \alpha^{(n)}\rangle}{\|\alpha' \wedge \alpha''\|\|\alpha' \wedge \alpha^{(n)}\|}, \sin\varphi = \langle D_2, B\rangle = -\langle D_1, N\rangle = \frac{v\det(\alpha^{(n)}, \alpha', \alpha'')}{\|\alpha' \wedge \alpha''\|.\|\alpha' \wedge \alpha^{(n)}\|}$$

In this situation, the relations between the elements $\{T, D_1, D_2, d_1, d_2, d_3\}$ and $\{T, N, B, \kappa, \tau\}$ are given following equalities $[9]$.

$$\begin{bmatrix} T \\ D_2 \\ D_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & \sin\varphi \\ 0 & -\sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} T \\ N \\ B \end{bmatrix} \text{ veya } \begin{bmatrix} T \\ N \\ B \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\varphi & -\sin\varphi \\ 0 & \sin\varphi & \cos\varphi \end{bmatrix} \begin{bmatrix} T \\ D_2 \\ D_1 \end{bmatrix}, \tag{1.7}$$

$$d_1 = \kappa\cos\varphi, \quad d_2 = -\kappa\sin\varphi, \quad d_3 = \tau + \frac{\varphi'}{v}, \tag{1.8}$$

The Darboux vector according to the Flc elements is given by the following relation $[11]$.

$$W = v(d_3 T - d_2 D_2 + d_1 D_1). \tag{1.9}$$

2. Pedal Curve and Smarandache Curves According to a Darboux Vector of a Frenet-like Frame

Definition 2.1. Let α be a nth degree regular polynomial curve and W be a Frenet-like Darboux vector in E^3. The geometric locus of perpendicular projection points onto the Darboux vector from a point $P \in E^3$ not on the curve is called $W-pedal$ curve of the curve α according to the point P.

The equation of the $W-pedal$ curve of the curve α is given by following relation.

$$\alpha_W(t) = \alpha(t) + \langle P - \alpha(t), W(t)\rangle W(t). \tag{2.1}$$

Specially, if we take the point P as origin, then we get $u(t) = -\langle \alpha(t), W(t)\rangle$. In this situation, the equation (2.1) is expressed as follows.

$$\alpha_W(t) = \alpha(t) + u(t)W(t). \tag{2.2}$$

Theorem 2.1 Let T_1, N_1 and B_1 denote the Frenet vectors of $W-pedal$ curve, respectively. The relations between the Frenet frame $\{T_1, N_1, B_1\}$ and Flc frame $\{T, D_2, D_1\}$ are given as follows:

$$\begin{aligned}
&T_1 = \omega_1 xT - \omega_1 yD_2 + \omega_1 zD_1,\\
&N_1 = B_1 \wedge T_1,\\
&B_1 = -\eta_1\left(yz' - zy' + xzvd_1 + yxvd_2 - y^2vd_3 - z^2vd_3\right)T + \eta_1\left(zx' - xz' + yzvd_1 - z^2vd_2 - x^2vd_2 + xyvd_3\right)D_2\\
&\quad + \eta_1\left(yx' - xy' + x^2vd_1 + y^2vd_1 - yzvd_2 - xzvd_3\right)D_1
\end{aligned}$$

where the coefficients x, y, z, ω_1 and η_1,

$$x = v + \left(uvd_3\right)',\ y = \left(uvd_2\right)',\ z = \left(uvd_1\right)',$$

$$\omega_1 = \frac{1}{\sqrt{\left(v + \left(uvd_3\right)'\right)^2 + \left(\left(uvd_2\right)'\right)^2 + \left(\left(uvd_1\right)'\right)^2}},$$

$$\eta_1 = \frac{1}{\sqrt{\begin{array}{l}\left(yz' - zy' + xzvd_1 + yxvd_2 - y^2vd_3 - z^2vd_3\right)^2 + \left(zx' - xz' + yzvd_1 - z^2vd_2 - x^2vd_2 + xyvd_3\right)^2\\ + \left(yx' - xy' + x^2vd_1 + y^2vd_1 - yzvd_2 - xzvd_3\right)^2\end{array}}}.$$

Proof: By taking the derivatives of the relation (2.2) and making necessary algebraic operations, we get following equalities.

$$\alpha_W' = xT - yD_2 + zD_1, \tag{2.3}$$

$$\alpha_W'' = \left(x' + yvd_1 - zvd_2\right)T + \left(-y' + xvd_1 - zvd_3\right)D_2 + \left(z' + xvd_2 - yvd_3\right)D_1, \tag{2.4}$$

$$\begin{aligned}
\alpha_W''' = &\left(\left(x' + yvd_1 - zvd_2\right)' - vd_1\left(-y' + xvd_1 - zvd_3\right) - vd_2\left(z' + xvd_2 - yvd_3\right)\right)T\\
&+\left(\left(-y' + xvd_1 - zvd_3\right)' + vd_1\left(x' + yvd_1 - zvd_2\right) - vd_3\left(z' + xvd_2 - yvd_3\right)\right)D_2\\
&+\left(\left(z' + xvd_2 - yvd_3\right)' + vd_2\left(x' + yvd_1 - zvd_2\right) + vd_3\left(-y' + xvd_1 - zvd_3\right)\right)D_1,
\end{aligned} \tag{2.5}$$

$$\begin{aligned}
\alpha_W' \wedge \alpha_W'' = &-\left(yz' - zy' + xzvd_1 + yxvd_2 - y^2vd_3 - z^2vd_3\right)T\\
&+\left(zx' - xz' + yzvd_1 - z^2vd_2 - x^2vd_2 + xyvd_3\right)D_2\\
&+\left(yx' - xy' + x^2vd_1 + y^2vd_1 - yzvd_2 - xzvd_3\right)D_1,
\end{aligned} \tag{2.6}$$

$$\begin{aligned}\det\left(\alpha_W',\alpha_W'',\alpha_W'''\right)=&-\left(yz'-zy'+xzvd_1+yxvd_2-y^2vd_3-z^2vd_3\right).\\&\left(\left(x'+yvd_1-zvd_2\right)'-vd_1\left(-y'+xvd_1-zvd_3\right)-vd_2\left(z'+xvd_2-yvd_3\right)\right)\\&+\left(zx'-xz'+yzvd_1-z^2vd_2-x^2vd_2+xyvd_3\right).\\&\left(\left(-y'+xvd_1-zvd_3\right)'+vd_1\left(x'+yvd_1-zvd_2\right)-vd_3\left(z'+xvd_2-yvd_3\right)\right)\\&+\left(yx'-xy'+x^2vd_1+y^2vd_1-yzvd_2-xzvd_3\right).\\&\left(\left(z'+xvd_2-yvd_3\right)'+vd_2\left(x'+yvd_1-zvd_2\right)+vd_3\left(-y'+xvd_1-zvd_3\right)\right),\end{aligned}\tag{2.7}$$

$$\left\|\alpha_W'\right\|=\sqrt{x^2+y^2+z^2},\tag{2.8}$$

$$\left\|\alpha_W'\wedge\alpha_W''\right\|=\sqrt{\begin{aligned}&\left(yz'-zy'+xzvd_1+yxvd_2-y^2vd_3-z^2vd_3\right)^2+\left(zx'-xz'+yzvd_1-z^2vd_2-x^2vd_2+xyvd_3\right)^2\\&+\left(yx'-xy'+x^2vd_1+y^2vd_1-yzvd_2-xzvd_3\right)^2\end{aligned}}\tag{2.9}$$

By substituting these expressions into the relation (1.1), this completes the proof.

Theorem 2.2 Let κ_1 and τ_1 denote the curvature and torsion functions of $W-pedal$ curve, respectively. Then, the curvatures are given by following equalities.

$$\kappa_1=\frac{\sqrt{\begin{aligned}&\left(yz'-zy'+xzvd_1+yxvd_2-y^2vd_3-z^2vd_3\right)^2+\left(zx'-xz'+yzvd_1-z^2vd_2-x^2vd_2+xyvd_3\right)^2\\&+\left(yx'-xy'+x^2vd_1+y^2vd_1-yzvd_2-xzvd_3\right)^2\end{aligned}}}{\left(x^2+y^2+z^2\right)\sqrt{x^2+y^2+z^2}},$$

$$\tau_1=\frac{\begin{aligned}&-\left(yz'-zy'+xzvd_1+yxvd_2-y^2vd_3-z^2vd_3\right).\left(\left(x'+yvd_1-zvd_2\right)'-vd_1\left(-y'+xvd_1-zvd_3\right)-vd_2\left(z'+xvd_2-yvd_3\right)\right)\\&+\left(zx'-xz'+yzvd_1-z^2vd_2-x^2vd_2+xyvd_3\right).\left(\left(-y'+xvd_1-zvd_3\right)'+vd_1\left(x'+yvd_1-zvd_2\right)-vd_3\left(z'+xvd_2-yvd_3\right)\right)\\&+\left(yx'-xy'+x^2vd_1+y^2vd_1-yzvd_2-xzvd_3\right).\left(\left(z'+xvd_2-yvd_3\right)'+vd_2\left(x'+yvd_1-zvd_2\right)+vd_3\left(-y'+xvd_1-zvd_3\right)\right)\end{aligned}}{\begin{aligned}&\left(yz'-zy'+xzvd_1+yxvd_2-y^2vd_3-z^2vd_3\right)^2+\left(zx'-xz'+yzvd_1-z^2vd_2-x^2vd_2+xyvd_3\right)^2\\&+\left(yx'-xy'+x^2vd_1+y^2vd_1-yzvd_2-xzvd_3\right)^2\end{aligned}}$$

Proof: If $(2.7),(2.8)$ and (2.9) are substituted into the relation (1.2), the curvatures are computed.

Corollary 2.1 The following relations exist between the Frenet vectors of $W-pedal$ curve and their derivatives.

$$T_1' = \mu_1\kappa_1 N_1,\ \ N_1' = \mu_1\left(-\kappa_1 T_1 + \tau_1 B_1\right),\ B_1' = -\mu_1\tau_1 N_1 \qquad (2.10)$$

where $\mu_1 = \|\alpha_T'\|$.

Definition 2.2 By taking the tangent and principal normal vectors of the $W-pedal$ curve as position vectors, a regular curve called the T_1N_1- Smarandache curve belonging to $W-pedal$ curve obtained from these vectors.

$$\alpha_1 = \frac{T_1 + N_1}{\sqrt{2}} \qquad (2.11)$$

Theorem2.3 Let T_{α_1}, N_{α_1} and B_{α_1} be the Frenet vectors of T_1N_1- Smarandache curve. Then, Frenet vectors are given as follows.

$$T_{\alpha_1} = \frac{-\kappa_1 T_1 + \kappa_1 N_1 + \tau_1 B_1}{\sqrt{2\kappa_1^2 + \tau_1^2}}$$
$$N_{\alpha_1} = B_{\alpha_1} \wedge T_{\alpha_1}$$
$$B_{\alpha_1} = \frac{-\left(\kappa_1 x_3 + \tau_1 x_2\right)T_1 + \left(\kappa_1 x_3 + \tau_1 x_1\right)N_1 - \left(\kappa_1 x_2 + \kappa_1 x_1\right)B_1}{\sqrt{\left(\kappa_1 x_3 + \tau_1 x_2\right)^2 + \left(\kappa_1 x_3 + \tau_1 x_1\right)^2 + \left(\kappa_1 x_2 + \kappa_1 x_1\right)^2}}$$

where the coefficients x_1, x_2 and x_3,

$$x_1 = -\frac{\mu_1^2\kappa_1^2 + \left(\mu_1\kappa_1\right)'}{\sqrt{2}}, x_2 = \frac{\left(\mu_1\kappa_1\right)' - \mu_1^2\left(\kappa_1^2 + \tau_1^2\right)}{\sqrt{2}}, x_3 = \frac{\mu_1^2\kappa_1\tau_1 + \left(\mu_1\tau_1\right)'}{\sqrt{2}}.$$

Proof: Let take the derivatives of the relation (2.11) and then, consider the relation (2.10). Following equalities are obtained.

$$\alpha_1' = \frac{\mu_1\left(-\kappa_1 T_1 + \kappa_1 N_1 + \tau_1 B_1\right)}{\sqrt{2}} \qquad (2.12)$$

$$\alpha_1'' = x_1 T_1 + x_2 N_1 + x_3 B_1 \qquad (2.13)$$

$$\alpha_1''' = \left(x_1' - \mu_1\kappa_1 x_2\right)T_1 + \left(x_2' + \mu_1 x_1 - \mu_1 x_3\right)N_1 + \left(x_3' + \mu_1\tau_1 x_2\right)B_1 \qquad (2.14)$$

$$\alpha_1' \wedge \alpha_1'' = -\frac{\mu_1\left(\kappa_1 x_3 + \tau_1 x_2\right)}{\sqrt{2}}T_1 + \frac{\mu_1\left(\kappa_1 x_3 + \tau_1 x_1\right)}{\sqrt{2}}N_1 - \frac{\mu_1\left(\kappa_1 x_2 + \kappa_1 x_1\right)}{\sqrt{2}}B_1 \qquad (2.15)$$

$$\det(\alpha_1', \alpha_1'', \alpha_1''') = \frac{\mu_1}{\sqrt{2}} \begin{pmatrix} (\kappa_1 x_3 + \tau_1 x_1)(x_2' + \mu_1 x_1 - \mu_1 x_3) \\ -(\kappa_1 x_3 + \tau_1 x_2)(x_1' - \mu_1 \kappa_1 x_2) - (\kappa_1 x_2 + \kappa_1 x_1)(x_3' + \mu_1 \tau_1 x_2) \end{pmatrix} \quad (2.16)$$

$$\|\alpha_1'\| = \frac{\mu_1}{\sqrt{2}} \sqrt{2\kappa_1^2 + \tau_1^2} \quad (2.17)$$

$$\|\alpha_1' \wedge \alpha_1''\| = \frac{\mu_1}{\sqrt{2}} \sqrt{(\kappa_1 x_3 + \tau_1 x_2)^2 + (\kappa_1 x_3 + \tau_1 x_1)^2 + (\kappa_1 x_2 + \kappa_1 x_1)^2} \quad (2.18)$$

If exps. (2.12), (2.15), (2.17) and (2.18) are substituted into (1.1), then, the curvatures are computed.

Theorem2.4 Let κ_{α_1} and τ_{α_1} denote the curvature and torsion functions of T_1N_1- Smarandache curve, respectively. Then, the curvatures are given by following equalities.

$$\kappa_{\alpha_1} = \frac{2\sqrt{(\kappa_1 x_3 + \tau_1 x_2)^2 + (\kappa_1 x_3 + \tau_1 x_1)^2 + (\kappa_1 x_2 + \kappa_1 x_1)^2}}{\mu_1^2 (2\kappa_1^2 + \tau_1^2)\sqrt{2\kappa_1^2 + \tau_1^2}},$$

$$\tau_{\alpha_1} = \frac{\sqrt{2}\left((\kappa_1 x_3 + \tau_1 x_1)(x_2' + \mu_1 x_1 - \mu_1 x_3) - (\kappa_1 x_3 + \tau_1 x_2)(x_1' - \mu_1 \kappa_1 x_2) - (\kappa_1 x_2 + \kappa_1 x_1)(x_3' + \mu_1 \tau_1 x_2)\right)}{\mu_1\left((\kappa_1 x_3 + \tau_1 x_2)^2 + (\kappa_1 x_3 + \tau_1 x_1)^2 + (\kappa_1 x_2 + \kappa_1 x_1)^2\right)}$$

Proof : By substituting these expressions (2.16),(2.17) and (2.18) into the relation (1.2), this completes the proof.

Definition 2.3 By taking the tangent and binormal vectors of the $W-pedal$ curve as position vectors, a regular curve called the T_1B_1- Smarandache curve belonging to $W-pedal$ curve obtained from these vectors.

$$\alpha_2 = \frac{T_1 + B_1}{\sqrt{2}}. \quad (2.19)$$

Theorem 2.5 Let T_{α_2}, N_{α_2} and B_{α_2} denote the Frenet vectors of T_1B_1- Smarandache curve, respectively. These vectors are given by following relations.

$$T_{\alpha_2} = N_1, N_{\alpha_2} = \frac{-\kappa_1 T_1 + \tau_1 B_1}{\sqrt{\kappa_1^2 + \tau_1^2}}, B_{\alpha_2} = \frac{\tau_1 T_1 + \kappa_1 B_1}{\sqrt{\kappa_1^2 + \tau_1^2}}.$$

Proof: By taking the derivatives of (2.19), and considering (2.10), following equalities are obtained.

$$\alpha_2' = \frac{\mu_1(\kappa_1 - \tau_1)N_1}{\sqrt{2}}, \tag{2.20}$$

$$\alpha_2'' = \frac{-\kappa_1\mu_1^2(\kappa_1 - \tau_1)T_1 + (\mu_1\kappa_1 - \mu_1\tau_1)' N_1 + \tau_1\mu_1^2(\kappa_1 - \tau_1)B_1}{\sqrt{2}}, \tag{2.21}$$

$$\alpha_2''' = \frac{\begin{array}{l}\left((-\kappa_1\mu_1^2(\kappa_1 - \tau_1))' - \kappa_1\mu_1(\mu_1\kappa_1 - \mu_1\tau_1)''\right)T_1 \\ +\left((\mu_1\kappa_1 - \mu_1\tau_1)'' - \mu_1^3(\kappa_1 - \tau_1)(\kappa_1^2 + \tau_1^2)\right)N_1 \\ +\left((\tau_1\mu_1^2(\kappa_1 - \tau_1))' + \tau_1\mu_1(\mu_1\kappa_1 - \mu_1\tau_1)''\right)B_1\end{array}}{\sqrt{2}}, \tag{2.22}$$

$$\alpha_2' \wedge \alpha_2'' = \frac{\mu_1^3(\kappa_1 - \tau_1)^2\tau_1 T_1 + \mu_1^3(\kappa_1 - \tau_1)^2\kappa_1 B_1}{2}, \tag{2.23}$$

$$\det(\alpha_2', \alpha_2'', \alpha_2''') = \frac{\mu_1^5(\kappa_1 - \tau_1)^3(\kappa_1\tau_1' - \tau_1\kappa_1')}{2\sqrt{2}}, \tag{2.24}$$

$$\|\alpha_2'\| = \frac{\mu_1(\kappa_1 - \tau_1)}{\sqrt{2}}, \tag{2.25}$$

$$\|\alpha_2' \wedge \alpha_2''\| = \frac{\mu_1^3(\kappa_1 - \tau_1)^2\sqrt{\kappa_1^2 + \tau_1^2}}{2} \tag{2.26}$$

If we substitute relations (2.20), (2.23), (2.25) and (2.26) into (1.1), then the proof is completed.

Theorem2.6 Let κ_{α_2} and τ_{α_2} denote the curvature and torsion functions of T_1B_1- Smarandache curve, respectively. Then, the curvatures are given by following equalities.

$$\kappa_{\alpha_2} = \frac{\sqrt{2\kappa_1^2 + 2\tau_1^2}}{(\kappa_1 - \tau_1)}, \quad \tau_{\alpha_2} = \frac{\sqrt{2}(\kappa_1\tau_1' - \tau_1\kappa_1')}{\mu_1(\kappa_1 - \tau_1)(\kappa_1^2 + \tau_1^2)}.$$

Proof: The proof is done upon substituting the relations (2.24), (2.25) ve (2.26) into (1.2)

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Definition 2.4 By taking the principal normal and binormal vectors of the $W-pedal$ curve as position vectors, a regular curve called the N_1B_1- Smarandache curve belonging to $W-pedal$ curve obtained from these vectors.

$$\alpha_3=\frac{N_1+B_1}{\sqrt{2}}. \tag{2.27}$$

Theorem 2.7 Let T_{α_3}, N_{α_3} and B_{α_3} be the Frenet vectors of N_1B_1- Smarandache curve. Then, Frenet vectors are given as follows.

$$T_{\alpha_3}=\frac{-\kappa_1T_1-\tau_1N_1+\tau_1B_1}{\sqrt{\kappa_1^2+2\tau_1^2}},$$
$$N_{\alpha_3}=B_{\alpha_3}\wedge T_{\alpha_3},$$
$$B_{\alpha_3}=\frac{-(\tau_1y_3+\tau_1y_2)T_1+(\kappa_1y_3+\tau_1y_1)N_1+(\tau_1y_1-\kappa_1y_2)B_1}{\sqrt{(\tau_1y_3+\tau_1y_2)^2+(\kappa_1y_3+\tau_1y_1)^2+(\tau_1y_1-\kappa_1y_2)^2}}$$

where the coefficients y_1,y_2 and y_3,

$$y_1=\frac{\mu_1^2\tau_1\kappa_1-(\mu_1\kappa_1)'}{\sqrt{2}},\ y_2=-\frac{\mu_1^2(\kappa_1^2+\tau_1^2)+(\mu_1\tau_1)'}{\sqrt{2}},\ y_3=\frac{(\mu_1\tau_1)'-\mu_1^2\tau_1^2}{\sqrt{2}}.$$

Proof: By taking the derivatives of (2.27), and considering (2.10), following equalities are obtained.

$$\alpha_3'=\frac{\mu_1(-\kappa_1T_1-\tau_1N_1+\tau_1B_1)}{\sqrt{2}}, \tag{2.28}$$

$$\alpha_3''=y_1T_1+y_2N_1+y_3B_1, \tag{2.29}$$

$$\alpha_3'''=(y_1'-\mu_1y_2\kappa_1)T_1+(y_2'+\mu_1y_1\kappa_1-\mu_1y_3\tau_1)N_1+(y_3'+\mu_1y_2\tau_1)B_1, \tag{2.30}$$

$$\alpha_3'\wedge\alpha_3''=\frac{\mu_1}{\sqrt{2}}((-(\tau_1y_3+\tau_1y_2)T_1+(\kappa_1y_3+\tau_1y_1)N_1+(\tau_1y_1-\kappa_1y_2)B_1)), \tag{2.31}$$

$$\det(\alpha_3',\alpha_3'',\alpha_3''')=\frac{\mu_1}{\sqrt{2}}\begin{pmatrix}(\tau_1y_1-\kappa_1y_2)(y_3'+\mu_1y_2\tau_1)\\-(\tau_1y_3+\tau_1y_2)(y_1'-\mu_1y_2\kappa_1)\\+(y_1'-\mu_1y_2\kappa_1)(y_2'+\mu_1y_1\kappa_1-\mu_1y_3\tau_1)\end{pmatrix} \tag{2.32}$$

$$\|\alpha_3'\| = \frac{\mu_1}{\sqrt{2}}\sqrt{\kappa_1^2 + 2\tau_1^2}, \tag{2.33}$$

$$\|\alpha_3' \wedge \alpha_3''\| = \frac{\mu_1}{\sqrt{2}}\sqrt{(\tau_1 y_3 + \tau_1 y_2)^2 + (\kappa_1 y_3 + \tau_1 y_1)^2 + (\tau_1 y_1 - \kappa_1 y_2)^2} \tag{2.34}$$

If we substitute the exps. (2.28), (2.31), (2.33) and (2.34) into (1.1), then we get the proof of the theorem.

Theorem2.8 Let κ_{α_3} and τ_{α_3} be the curvature and torsion functions of N_1B_1- Smarandache curve, respectively. Then, the curvatures are given as follows.

$$\kappa_{\alpha_3} = \frac{2\sqrt{(\tau_1 y_3 + \tau_1 y_2)^2 + (\kappa_1 y_3 + \tau_1 y_1)^2 + (\tau_1 y_1 - \kappa_1 y_2)^2}}{\mu_1^2(\kappa_1^2 + 2\tau_1^2)\sqrt{\kappa_1^2 + 2\tau_1^2}},$$

$$\tau_{\alpha_3} = \frac{\sqrt{2}\left((\tau_1 y_1 - \kappa_1 y_2)(y_3' + \mu_1 y_2\tau_1) - (\tau_1 y_3 + \tau_1 y_2)(y_1' - \mu_1 y_2\kappa_1) + (y_1' - \mu_1 y_2\kappa_1)(y_2' + \mu_1 y_1\kappa_1 - \mu_1 y_3\tau_1)\right)}{\mu_1\left((\tau_1 y_3 + \tau_1 y_2)^2 + (\kappa_1 y_3 + \tau_1 y_1)^2 + (\tau_1 y_1 - \kappa_1 y_2)^2\right)}$$

Proof: Substituting the expressions (2.32), (2.33) and (2.34) into relation (1.2) completes the proof.

Definition 2.5 By taking the tangent, principal normal and binormal vectors of the $W-pedal$ curve as position vectors, a regular curve called the $T_1N_1B_1-$ Smarandache curve belonging to $W-pedal$ curve obtained from these vectors.

$$\alpha_4 = \frac{T_1 + N_1 + B_1}{\sqrt{3}} \tag{2.35}$$

Theorem 2.9 Let T_{α_4}, N_{α_4} and B_{α_4} be the Frenet vectors of $T_1N_1B_1-$ Smarandache curve. Then, Frenet vectors are given as follows.

$$T_{\alpha_4} = \frac{-\kappa_1 T_1 + (\kappa_1 - \tau_1)N_1 + \tau_1 B_1}{\sqrt{2\kappa_1^2 - 2\kappa_1\tau_1 + 2\tau_1^2}},$$

$$N_{\alpha_4} = B_{\alpha_4} \wedge T_{\alpha_4},$$

$$B_{\alpha_4} = \frac{(z_3\kappa_1 - z_3\tau_1 - \tau_1 z_2)T_1 + (\tau_1 z_1 + \kappa_1 z_3)N_1 - (\kappa_1 z_1 - \tau_1 z_1 + \kappa_1 z_2)B_1}{\sqrt{(z_3\kappa_1 - z_3\tau_1 - \tau_1 z_2)^2 + (\tau_1 z_1 + \kappa_1 z_3)^2 + (\kappa_1 z_1 - \tau_1 z_1 + \kappa_1 z_2)^2}}$$

where the coefficients z_1, z_2 and z_3.

$$z_1 = -\frac{(\mu_1\kappa_1)' + \mu_1^2\kappa_1(\kappa_1 - \tau_1)}{\sqrt{3}}, z_2 = \frac{(\mu_1\kappa_1 - \mu_1\tau_1)' - \mu_1^2(\kappa_1^2 + \tau_1^2)}{\sqrt{3}}, z_3 = \frac{(\mu_1\tau_1)' + \mu_1^2\tau_1(\kappa_1 - \tau_1)}{\sqrt{3}}.$$

Proof: Let take the derivatives (2.35) and consider (2.10). Following equalities are given.

$$\alpha_4' = \frac{\mu_1(-\kappa_1 T_1 + (\kappa_1 - \tau_1)N_1 + \tau_1 B_1)}{\sqrt{3}} \tag{2.36}$$

$$\alpha_4'' = z_1 T_1 + z_2 N_1 + z_3 B_1 \tag{2.37}$$

$$\alpha_4''' = (z_1' - z_2\mu_1\kappa_1)T_1 + (z_2' + z_1\mu_1\kappa_1 - z_3\mu_1\tau_1)N_1 + (z_3' + z_2\mu_1\tau_1)B_1 \tag{2.38}$$

$$\alpha_4' \wedge \alpha_4'' = \frac{\mu_1}{\sqrt{3}}\left((z_3\kappa_1 - z_3\tau_1 - \tau_1 z_2)T_1 + (\tau_1 z_1 + \kappa_1 z_3)N_1 - (\kappa_1 z_1 - \tau_1 z_1 + \kappa_1 z_2)B_1\right) \tag{2.39}$$

$$\det(\alpha_4', \alpha_4'', \alpha_4''') = \frac{\mu_1}{\sqrt{3}}\begin{pmatrix}(z_3\kappa_1 - z_3\tau_1 - \tau_1 z_2)(z_1' - z_2\mu_1\kappa_1) + (\tau_1 z_1 + \kappa_1 z_3)(z_2' + z_1\mu_1\kappa_1 - z_3\mu_1\tau_1) \\ -(\kappa_1 z_1 - \tau_1 z_1 + \kappa_1 z_2)(z_3' + z_2\mu_1\tau_1)\end{pmatrix} \tag{2.40}$$

$$\|\alpha_4'\| = \frac{\sqrt{6}\mu_1}{3}\sqrt{\kappa_1^2 - \kappa_1\tau_1 + \tau_1^2} \tag{2.41}$$

$$\|\alpha_4' \wedge \alpha_4''\| = \frac{\mu_1}{\sqrt{3}}\sqrt{(z_3\kappa_1 - z_3\tau_1 - \tau_1 z_2)^2 + (\tau_1 z_1 + \kappa_1 z_3)^2 + (\kappa_1 z_1 - \tau_1 z_1 + \kappa_1 z_2)^2} \tag{2.42}$$

Substituting the expressions (2.36), (2.39), (2.41) and (2.42) into relation (1.1) completes the proof.

Theorem 2.10 Let κ_{α_4} and τ_{α_4} denote the curvature and torsion functions of $T_1N_1B_1$ – Smarandache curve, respectively. Then, the curvatures are given by following equalities.

$$\kappa_{\alpha_4} = \frac{3\sqrt{2}\sqrt{(z_3\kappa_1 - z_3\tau_1 - \tau_1 z_2)^2 + (\tau_1 z_1 + \kappa_1 z_3)^2 + (\kappa_1 z_1 - \tau_1 z_1 + \kappa_1 z_2)^2}}{4\mu_1^2(\kappa_1^2 - \kappa_1\tau_1 + \tau_1^2)\sqrt{\kappa_1^2 - \kappa_1\tau_1 + \tau_1^2}},$$

$$\tau_{\alpha_4}=\frac{\sqrt{3}\left(\begin{array}{l}\left(z_3\kappa_1-z_3\tau_1-\tau_1z_2\right)\left(z_1'-z_2\mu_1\kappa_1\right)+\left(\tau_1z_1+\kappa_1z_3\right)\left(z_2'+z_1\mu_1\kappa_1-z_3\mu_1\tau_1\right)\\-\left(\kappa_1z_1-\tau_1z_1+\kappa_1z_2\right)\left(z_3'+z_2\mu\tau_1\right)\end{array}\right)}{\mu_1\left(\left(z_3\kappa_1-z_3\tau_1-\tau_1z_2\right)^2+\left(\tau_1z_1+\kappa_1z_3\right)^2+\left(\kappa_1z_1-\tau_1z_1+\kappa_1z_2\right)^2\right)}.$$

Proof: Substituting the expressions (2.40), (2.41) and (2.42) into relation (1.2), it completes the proof.

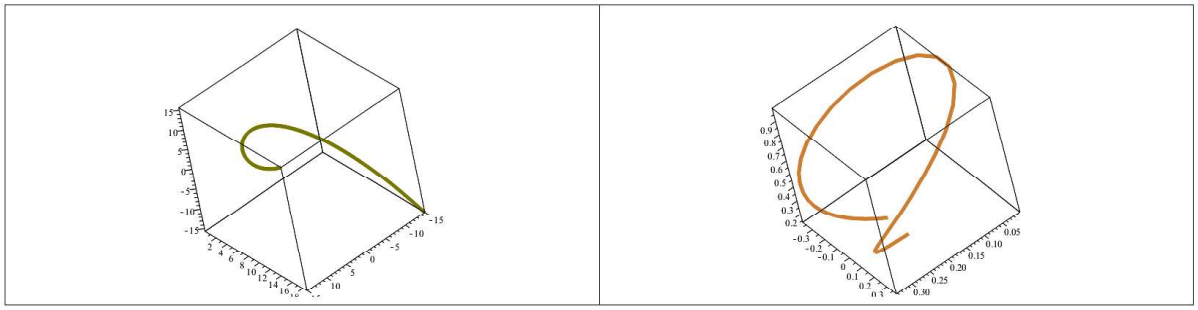
Example 2.1 Let think about the curve parametrized as $\alpha(t)=\left(6t,3t^2,t^3\right)$. The Flc frame and Darboux vector are given as follows.

$$\begin{cases}T=\dfrac{\left(2,2t,t^2\right)}{t^2+2},D_2=\dfrac{\left(-t^2,-t^3,2t^2+2\right)}{\left(t^2+2\right)\sqrt{t^2+1}},D_1=\dfrac{(t,-1,0)}{\sqrt{t^2+1}},\\ d_1=\dfrac{2t}{3\left(t^2+2\right)^2\sqrt{t^2+1}},d_2=\dfrac{-2}{3\left(t^2+2\right)^2\sqrt{t^2+1}},d_3=\dfrac{t^2}{3\left(t^2+2\right)^2\left(t^2+1\right)},\end{cases}$$

$$W=\frac{1}{\left(t^2+2\right)^2\left(t^2+1\right)}\left(2t^2,-2t,\left(t^2+2\right)^2\right)$$

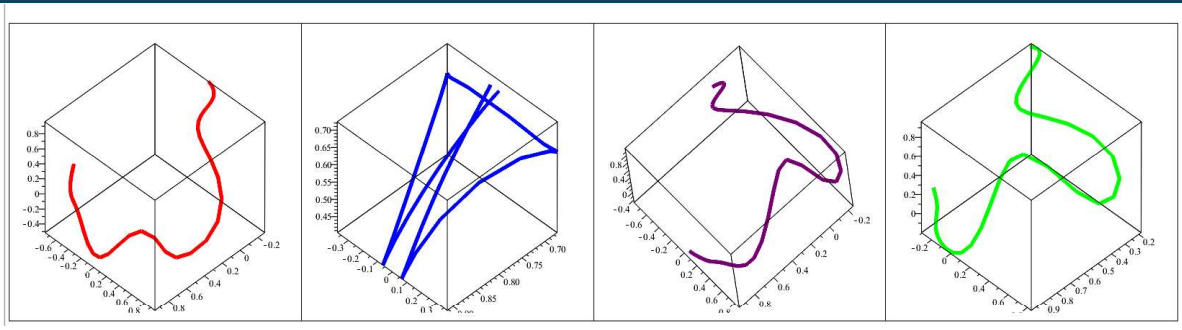
The equation of the $W-pedal$ curve from the relation (2.2) is given by the following equality.

$$\alpha_W(t)=\left(6t,3t^2,t^3\right)-\frac{t^3\left(t^4+4t^2+10\right)}{\left(t^2+2\right)^4\left(t^2+1\right)^2}\left(2t^2,-2t,\left(t^2+2\right)^2\right)$$



The curve α $W-pedal$ Pedal curve of the curve α

Fig. 1 The curve α and $W-pedal$ curve of the curve α



T_1N_1 −Smarandache curve T_1B_1 −Smarandache curve N_1B_1 −Smarandache curve $T_1N_1B_1$ −Smarandache curve

Fig. 2 The Smarandache Curves obtained from Frenet vectors of W −pedal curve of the curve α

REFERENCES

[1] A.T. Ali, Special Smarandache Curves in the Euclidean Space, International Journal of Mathematical Combinatorics, 2, 30-36. 2010.
[2] A. Gray, E. Abbena and S. Salamon, Modern Differential Geometry of Curves and Surfaces with Mathematica, 3rd ed. Chapman and Hall/CRC: Boca Raton, FL, USA, 1016 pages, 2006.
[3] A.Y. Ceylan, M. Kara, On Pedal and Contrapedal Curves of Bezier Curves, Konuralp Journal of Mathematics, 9 (2), 217-221. 2021.
[4] D. Canlı, S. Şenyurt, F.E. Kaya, and L. Grilli, The Pedal Curves Generated by Alternative Frame Vectors and Their Smarandache Curves, Symmetry 2024, 16(8), 1012; 28 pages, https://doi.org/10.3390/sym16081012.
[5] E. As, A. Sarioğlugil, On the Pedal Surfaces of 2-d Surfaces with the Constant Support Function in E^4, Pure Mathematical Sciences, 4(3), 105-120. 2015.
[6] F. Ertem Kaya, Differential Geometric Aspects of Pedal Curves on Surfaces, Journal of Mathematics, Wiley, Volume 2024, Article ID 6237091, 11 pages, https://doi.org/10.1155/2024/6237091.
[7] M. P. do Carmo, Differential Geometry of Curves and Surfaces, Prentice Hall, Englewood Cliffs, NJ, 1976.
[8] M. Turgut, S. Yılmaz, Smarandache Curves in Minkowski Space-Time, International Journal of Mathematical Combinatorics, 3, 51-55. 2008.
[9] M. Dede, A new representation of tubular surfaces, Houston Journal of Mathematics, 45(2018), 707–720. 2019.
[10] M. Dede, On Polynomial Space Curves with Flc-frame, Turkish Journal of Mathematics and Computer Science, 15(2), 414-422. 2023.
[11] M. Dede, Why Flc-Frame is Better than Frenet Frame on Polynomial Space Curves?, Mathematical Sciences and Applications E-Notes, 10 (4) 190-198. 2022.
[12] N. Kuruoğlu, A. Sarıoğlugil, On the characteristic properties of the a-Pedal surfaces in the Euclidean space E^3. Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistics, 42, 19-25. 1993.
[13] O.O. Tuncer, H. Ceyhan, İ. Gök, F. N. Ekmekci, Notes on pedal and contrapedal curves of fronts in the Euclidean plane, Mathematical Methods in the Applied Sciences, 41(13), 5096–5111. 2018.
[14] Y. Li, Donghe Pei, Pedal curves of fronts in the sphere, Journal of Nonlinear Sciences and Applications, 9(3), 836-844. 2016.
[15] Y. Li, Pei D., Pedal curves of frontals in the Euclidean plane, Mathematical Methods in the Applied Sciencesi, 41(5), 1988-1997. 2018.
[16] Y. Li, O. Oğulcan Tuncer, On (contra)pedals and (anti)orthotomics of frontals in de Sitter 2-space, Mathematical Methods in the Applied Sciences, 46(9), 11157-11171. 2023.

[17] S. Şenyurt, S. Sivas, An Application of Smarandache Curve, Ordu University Journal of Science and Technology, 3(1), 46-60. 2013.
[18] S. Şenyurt, D. Canlı, and K.H. Ayvacı, Associated Curves From A Different Point of View in E^3, Communications Faculty of Science University of Ankara Series A1 Mathematics and Statistics, 71(3), 826–845. 2022.
[19] S. Şenyurt, K. Eren, and K.H. Ayvacı, A Study on Inextensible Flows of Polynomial Curves with Flc Frame, Applications and Applied Mathematics: An International Journal, 17(1), 123 – 133. 2022.
[20] S. Şenyurt, K.H. Ayvacı, and D. Canlı, Smarandache Curves According to Flc-frame in Euclidean 3-space, Fundamentals of Contemporary Mathematical Sciences, 4(1), 16-30. 2023.
[21] S. Şenyurt, K.H. Ayvacı, and D. Canlı, Special Smarandache Ruled Surfaces According to Flc frame in E^3, Applications and Applied Mathematics: An International Journal,18(1), 18 pages, 2023.
[22] S. Şenyurt, K.H. Ayvacı, and D. Canlı, Some Characterizations of Spherical Indicatrix Curves Generated by Flc Frame, Turkish Journal of Mathematics and Computer Science, 13(2), 379-387. 2021.
[23] S. Şenyurt, K. Eren, and K.H. Ayvacı, Characterizations of tzitzeica curves using FLC frame, Sigma Journal of Engineering and Natural Sciences, 42(1), 37-41. 2024.
[24] S. Şenyurt, F. Ertem Kaya, and D. Canlı, Pedal curves obtained from Frenet vector of a space curve and Smarandache curves belonging to these curves, AIMS Mathematics, 9(8), 20136-20162, 2024.
[25] S. Şenyurt, F. Ertem Kaya, Pedal Curves and Applications According to a Vector lying on the Osculator Plane, Proceeding Book of 2nd International Conference on Pioneer and Innovative Studies ICPIS 2025, All Sciences Academy, 31-40. ISBN: 978-625-5954-58-9, Virtual Conference date: January 30-31, 2025, Konya, TÜRKİYE, Published date: February 6, 2025.
[26] S. Şenyurt, F. Ertem Kaya, Pedal Curves and Applications According to a Vector lying on the Normal Plane, Proceeding Book of 2nd International Conference on Recent and Innovative Results in Engineering and Technology ICRIRET 2025, All Sciences Academy, 16-26. ISBN: 978-625-5954-62-6, Virtual Conference date: February 11-12, 2025, Konya, TÜRKİYE, Published date: February 20, 2025.
[27] S. Şenyurt, F. Ertem Kaya, Pedal Curves and Applications According to a Vector lying on the Rectifying Plane, Proceeding Book of 4th International Conference on Contemporary Academic Research ICCAR 2025, All Sciences Academy, 21-31. ISBN: 978-625-5954-79-4, Virtual Conference date: February 22-23, 2025, Konya, TÜRKİYE, Published date: March 2, 2025.
[28] S. Şenyurt, F. Ertem Kaya, Pedal Curves and Applications Obtained From A Polynomial Curve According to a Vector lying on the $sp\{T, D_2\}-$ Plane, Proceeding Book of 4th International Conference on Recent Academic Studies ICRAS 2025, All Sciences Academy, 14-25. ISBN: 978-625-5954-91-6, Virtual Conference date: March 4-5, 2025, Konya, TÜRKİYE, Published date: March 15, 2025.
[29] S. Şenyurt, F. Ertem Kaya, Pedal Curves and Applications According to a Vector lying on $sp\{D_2, D_1\}-$ Plane, Proceeding Book of 6th International Conference on Innovative Academic Studies ICIAS 2025, All Sciences Academy, 1-12, ISBN: 978-625-5900-02-9, Virtual Conference date: March 12-13, 2025, Konya, TÜRKİYE, Published date: March 20, 2025.
[30] S. Şenyurt, F. Ertem Kaya, Pedal Curves and Applications Obtained from A Polynomial Curve according to A Vector Lying on $sp\{T, D_1\}-$Plane, Proceeding Book of 6th International Conference on Scientific and Academic Research ICSAR 2025, All Sciences Academy, 1-12, ISBN: 978-625-5900-08-01, Virtual Conference date: March 19-20, 2025, Konya, TÜRKİYE, Published date: March 25, 2025.
[31] S. Şenyurt, F. Ertem Kaya, Pedal Curves Obtained from the Darboux Vector of the Alternative Frame and Its Applications, Proceeding Book of 3rd International Conference on Trends in Advanced Research ICTAR 2025, All Sciences Academy, 36-46, ISBN: 978-625-5900-21-0, Virtual Conference date: April 04-05, 2025, Konya, TÜRKİYE, Published date: April 25, 2025.

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